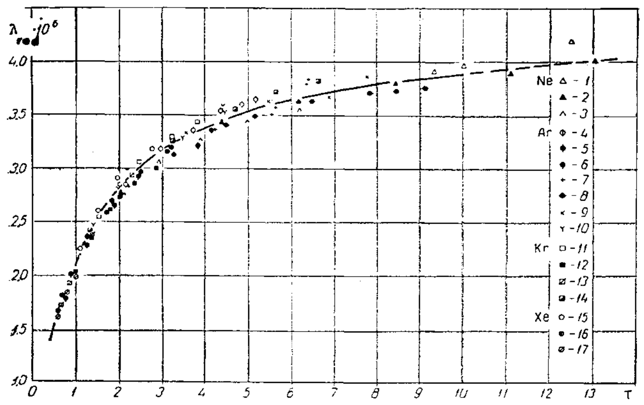

Fig. 1 Generalization of heat conductivity of monatomic gases according to data of: 1, 4, 11, 15—Zaitseva[5]; 2, 5, 12, 16—Kannuluik and Carman[6]; 3, 6, 13, 17—Keyes[7]; 7—Rothman and Bromley[10]; 8, 14—Schäfer and Reiter[8]; 9—Vines[9]; 10—Tsederberg, Popov, and Morozova[11]

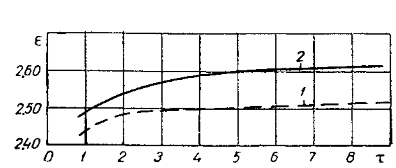

Fig. 2 Coefficient ε as a function of the reduced temperature: 1) according to the modified Enskog equation (12); 2) according to the results of the author's generalization

basis of our generalization, is roughly twice that given by Enskog's equation.

In conclusion we should point out that the equation

$$\lambda = \lambda_0(T/T_0)^n \tag{13}$$

(where λ_0 is the heat conductivity for $T_0 = 273°K$, and n is an exponent depending on the nature of the gas), which is at present widely used for extrapolation, is obviously less accurate than expression (4) or an equation of the Sutherland type. The latter give a qualitatively superior representation of the dependence of the heat conductivity on temperature. Thus, for example, from an analysis of experimental material (up to 515°C) for argon, Zaitseva[5] has found $\lambda_0 = 142$ kcal/m·hr·deg and $n = 0.80$. Then Eq. (13), with $t = 1000°C$, gives $\lambda = 486$ kcal/m·hr·deg, but (3) and (4) give only 433 and 443 kcal/m·hr·deg, respectively. Here it is assumed that $\eta = 616\cdot10^{-6}$ g/cm·sec, according to experimental data, and $\epsilon = 2.68$, according to an equation of type (5). As shown by the experimental data (Fig. 1), for $t = 1000°C$, the heat conductivity of argon is equal to 437 kcal/m·hr·deg—i.e., extrapolation from Eq. (13) leads to an error of ~10%, whereas Eqs. (3) and (4) give good agreement with experiment. An analogous result is obtained for krypton, which has been investigated experimentally up to a temperature of 1100°C.[8]

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Comment on "Equations of the Precessional Theory of Gyroscopes"

Norman E. French*
Litton Systems (Canada) Ltd., Rexdale, Ontario, Canada

Nomenclature

A,B,C = moments of inertia of a rigid body about its principal axes a,b,c
ω = component of angular velocity of a rigid body
Ω = component of angular velocity of a system of orthogonal axes x,y,z
M = component of sum of the moments of forces applied to a rigid body
f = frequency

Subscripts

a,b,c = principal axes of inertia of a rigid body
x,y,z = system of orthogonal axes; axis z is identical with axis c

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* Senior Engineer, Engineering Department.

In a translation of a paper by L. I. Kuznetsov,† three equations are given which, when expressed in symbols more familiar to Western readers, take the form

$$\begin{aligned} M_x &= A(\dot{\Omega}_x - \Omega_z\Omega_y) + C\omega_z\Omega_y \\ M_y &= A(\dot{\Omega}_y + \Omega_z\Omega_x) - C\omega_z\Omega_x \\ M_z &= C\dot{\omega}_z \end{aligned} \tag{1}$$

These are closely related to Euler's equations but are not usually so called.

The author then introduces approximate equations which may be called the equations of precession. In the nomenclature used here these equations are

$$M_x = C\omega_z\Omega_y \qquad M_y = -C\omega_z\Omega_x \qquad M_z = C\dot{\omega}_z \tag{2}$$

Euler's equations are valid for any rigid body. The body need not be dynamically symmetrical about any axis. All vectors are referred to a set of orthogonal axes a,b,c which are fixed in the body and which coincide with the principal axes of inertia.

† Kuznetsov, L. I., "Equations of the precessional theory of gyroscopes," AIAA J. **1**, 271–274 (1963).

Euler's equations are

$$\begin{aligned} M_a &= A\dot{\omega}_a - (B - C)\omega_b\omega_c \\ M_b &= B\dot{\omega}_b - (C - A)\omega_c\omega_a \\ M_c &= C\dot{\omega}_c - (A - B)\omega_a\omega_b \end{aligned} \tag{3}$$

Equations (1) are valid only for a rigid body which is dynamically symmetrical about an axis z. Axes x and y are any pair of orthogonal axes perpendicular to z. Axis z is fixed in the body, but axes x and y are not. The z component of the angular velocity of the system of axes x,y,z is arbitrary.

It is convenient to select the axes x and y such that the z component of angular velocity of the system of axes x,y,z is zero or negligible. By setting $\Omega_z = 0$ in Eqs. (1) one obtains for this special case

$$\begin{aligned} M_x &= A\dot{\Omega}_x + C\omega_z\Omega_y \\ M_y &= A\dot{\Omega}_y - C\omega_z\Omega_x \\ M_z &= C\dot{\omega}_z \end{aligned} \tag{4}$$

Kuznetsov drops the terms resulting from Ω_z at his Eq. (5). This is possible because the axes x and y were so chosen that Ω_z remains small. The analysis could therefore have commenced with the foregoing equations (4), rather than with Eqs. (1).

The author considers some special cases of the motion of the point of support of a pendulous vertical gyroscope. Two cases are discovered where the equations of precession (2) yield results at variance with the results obtained by Eqs. (1). The two cases are as follows:

1) The point of support performs harmonic oscillations along a great circle with frequency $\omega_2/2\pi$.

2) The point of support moves along a horizontal circle of radius ρ with speed v such that $v/\rho = \omega_2$.

In Kuznetsov's Eq. (17), ω_2 is given, which in the nomenclature used here becomes $\omega_2 = (C/A)\omega_z$.

In both cases, the point of support is subjected to forced oscillations, at a frequency f given by

$$f = \omega_2/2\pi = \frac{1}{2\pi}\left\{\frac{C}{A}\right\}\omega_z \tag{5}$$

The pendulous mass will respond to the forced oscillations by applying sinusoidal torques to the gyroscope at frequency f.

The frequency f is, however, the nutation frequency of the gyroscope. This may be seen readily by equating M_x and M_y in Eqs. (4) to zero, and solving for Ω_x and Ω_y.

Normally, nutation may be neglected, since it is rapidly damped by viscous forces in the suspension. The rotational motion of the gyroscope may be obtained by solving the equations of precession. However, if the gyroscope is subjected to sinusoidal torquing at or near the nutation frequency, a nutation of expanding amplitude will result. It is not surprising that the equations of precession then fail to provide an adequate description of the rotational motion of the gyroscope.

Digest of Translated Russian Literature

The following abstracts have been selected by the Editor from translated Russian journals supplied by the indicated societies and organizations, whose cooperation is gratefully acknowledged. Information concerning subscriptions to the publications may be obtained from these societies and organizations. Note: Volumes and numbers given are those of the English translations, not of the original Russian.

SOVIET PHYSICS—JETP (Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki). Published by American Institute of Physics, New York

Volume 13, Number 2, August 1961

Three-Body Problem with Short-Range Forces, G. S. Danilov, pp. 349–355.

Skornyakov and Ter-Martirosyan have derived equations for the determination of the wave function of a system of three identical particles in the limiting case of zero-range forces. These equations allow one to express the scattering amplitude for the scattering of a neutron on a deuteron with the total spin $S = \frac{3}{2}$ for the system in terms of the parameters of the two-particle problem. Analogous calculations for the case $S = \frac{1}{2}$, however, have not been successful.

For the total spin $S = \frac{1}{2}$ the wave function of the system does not vanish when the distances between all particles become zero, as in the case when $S = \frac{3}{2}$. It will be shown in the present paper that the forementioned equations have a non-unique solution in this case. We shall also see that the wave functions for different energies are proportional to one another in the region where the three particles are sufficiently close to each other. We must therefore choose that solution of the equations of Skornyakov and Ter-Martirosyan which guarantees that the wave functions for different energies are proportional to each other in the forementioned region. This requirement allows us to choose a unique solution for an arbitrary energy if the wave function for a single value of the energy is known. For this wave function one can, for example, take the wave function of tritium. The determination of this wave function requires the knowledge of the binding energy of tritium. To solve the three-body problem with short range forces, we thus require one more parameter in addition to the parameters of the two-body problem, as, for example, the binding energy of tritium.

The idea that the non-uniqueness of the solution of the equations of Skornyakov and Ter-Martirosyan can be removed by introducing an experimental parameter is due to Gribov.

In the present paper we explain why the equation of Skornyakov and Ter-Martirosyan for three identical spinless particles has no unique solution. We also determine the behavior of the wave function in the region where the distances between all particles are close to zero. The rigorous proof of the non-uniqueness of the solution of the equation of Skornyakov and Ter-Martirosyan is given. We prove that the wave functions for different energies are proportional to one another in the region where the distances between all particles are close to zero. We derive for the wave function of three identical spinless particles an equation with a unique solution. In this form the equation of Skornyakov and Ter-Martirosyan can be solved numerically. We consider the same problem, but take in account the spin and the isotopic spin of the particles.

Physical Meaning of Negative Probabilities, J.-P. Vigier and Ya. P. Terletskii, pp. 356–359.

It is shown that the calculation of the statistical averages of a series of physical quantities can be carried out with the help of the distribution function instead of the probability density. The